

ASSIGNMENT 1, DUE MONDAY 18 SEPTEMBER

1. Let $\mathbb{Q}(\sqrt{2})$ denote the set of all real numbers of the form $a + b\sqrt{2}$, where a and b are rational numbers. Show that $\mathbb{Q}(\sqrt{2})$ has the structure of a field under the usual operations of addition and multiplication in \mathbb{R} .
2. Let n be a positive integer. Show that \mathbb{Z}/n is a field if and only if n is prime. You may use the following without proving it: *If a and b are integers, then the equation $ax + by = 1$ has a solution if and only if $\gcd(a, b) = 1$.*

3. Consider the following system of linear equations.

$$2x_1 + x_3 + 3x_4 = 2$$

$$x_1 + x_2 + x_4 = 5$$

$$4x_2 + 2x_3 + x_4 = 9$$

$$3x_1 + x_2 + x_3 = 7$$

- (a) Find the coefficient matrix and the augmented matrix for this system.
- (b) Use row reduction to find all solutions to the above system, detailing all steps. Indicate when the matrix is in echelon form and reduced echelon form. Identify the basic and free variables.
- (c) Check your answer.

4. Consider the following system of linear equations.

$$6x_1 + 12x_2 + 3x_4 = 12$$

$$x_1 + 2x_2 + 2x_3 + 3x_4 = 13$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 18$$

$$2x_1 + 4x_2 + 3x_3 + 4x_4 = 19$$

- (a) Find the coefficient matrix and the augmented matrix for this system.
- (b) Use row reduction to find all solutions to the above system, detailing all steps. Indicate when the matrix is in echelon form and reduced echelon form. Identify the basic and free variables.
- (c) Check your answer.

5. Use matrix row reduction to find all values of $a \in \mathbb{R}$ for which the following system of linear equations is consistent.

$$x_1 + ax_2 + x_3 = 2$$

$$x_2 + 2x_3 = 0$$

$$3x_1 + x_3 = 1$$

6. Convert the systems of linear equations in questions 3 and 4 to vector equations and matrix equations.