ASSIGNMENT 1, DUE WEDNESDAY 31 JANUARY

1. Let P, Q and R be propositions. Prove that

$$P \Longrightarrow (Q \vee R) \;\equiv\; (P \wedge \neg Q) \Longrightarrow R$$

using
(a) truth tables;
(b) known equivalences presented in the book or in class.

2. Show that the propositional calculus can be expressed purely in terms of the two connectives $\neg$ and $\wedge$ by finding propositional forms that are equivalent to $P \vee Q$, $P \Rightarrow Q$ and $P \Leftrightarrow Q$, and only involve $\neg$ and $\wedge$.

3. Find a propositional form equivalent to $P \wedge Q$ that only involves $\neg$ and $\Rightarrow$. By the previous exercise, this shows that the propositional calculus can be expressed purely in terms of the logical connectives $\neg$ and $\Rightarrow$.

4. The *NAND* connective of P and Q is the propositional form $P \mid Q$ with truth table

P	Q	$P \mid Q$
T	T	F
F	T	T
T	F	T
F	F	T

(hence the name "Not AND"). Show that the propositional calculus can be expressed using only the NAND connective. (Exercise 2 might make things easier. Also, start by NANDing a proposition P with itself.)

5. In formal logic, the definition of a theorem is more restrictive than the one we'll be learning in this class. There, a *theorem* is a proposition P for which there is a finite list of propositions

$$P_0$$
$$P_1$$
$$\vdots$$
$$P_{n-1}$$
$$P_n$$

such that P_n is P. Moreover, for each index $0 \le i \le n$ the proposition P_i must satisfy one of the following.

- P_i is a tautology.
- There exist indices $j < i$ and $k < i$ for which P_k is the proposition $P_j \Rightarrow P_i$.

(The second condition is called *modus ponens*.) The list $P_0, \ldots, P_n$ is then called a *proof* of P.

Show that if P and Q are theorems, then $P \wedge Q$ is a theorem.

6. Consider the statements

$C(x)$: "x is a car"

$G(x)$: "x runs on gasoline"

$E(x)$: "x runs on electricity"

Write the following sentences using quantifiers and logical connectives in the universe $\mathcal{U}$ of all objects, and then in the universe $\mathcal{V}$ of all cars.

(a) "All cars run on gasoline."
(b) "Some cars run on electricity."
(c) "Some cars are hybrid."
(d) "Cars that do not run on gasoline must run on electricity."

Use your answers to find useful denials of the above sentences (in English prose).