ASSIGNMENT 10, DUE FRIDAY 20 APRIL

1. Of the properties

reflexive, *symmetric*, *transitive*

find (if they exist) examples of relations on sets that satisfy none of them, each of them but not the other two, and two of them but not the third.

2. Let A be a set and R a relation on A. Prove that if R is symmetric and transitive, and if $\operatorname{Dom} R = A$, then R is an equivalence relation.

3. How many equivalence relations are there on the set $\{0, 1, 2, 3\}$?

4. Consider the partition

$$\mathscr{P} = \{\{0\}, \{-1, 1\}, \{-2, 2\}, \{-3, 3\}, \ldots\}$$

of $\mathbb{Z}$. Describe, as concretely as possible, the equivalence relation R on $\mathbb{Z}$ such that $\mathbb{Z}/R = \mathscr{P}$.

5. Let $\mathscr{P}$ and $\mathscr{Q}$ be partitions of a set A. We say that $\mathscr{P}$ is *finer* than $\mathscr{Q}$ and that $\mathscr{Q}$ is *coarser* than $\mathscr{P}$ if the following condition holds.

For each $X \in \mathscr{P}$ there exists $Y \in \mathscr{Q}$ with $X \subseteq Y$.

Find the equivalence relations R and S on A such that A/R is as coarse as possible and A/S is as fine as possible.

6. Write down the addition and multiplication tables for $\mathbb{Z}/6$.

7. Let n be a composite integer. Prove that there always exist elements $\overline{a}$ and $\overline{b}$ in $\mathbb{Z}/n$ such that $\overline{a} \neq \overline{0}$ and $\overline{b} \neq \overline{0}$, but $\overline{a} \times_n \overline{b} = \overline{0}$.