ASSIGNMENT 11, DUE MONDAY 30 APRIL

1. Let $f\colon A \to B$ be a function. One may check (you do not need to) that

$$x \, R \, y \qquad \text{iff} \qquad f(x) = f(y)$$

defines an equivalence relation on A. Now consider the relation

$$F = \{(x/R, f(y)) \mid x, y \in A \text{ and } y \in x/R\}$$

from A/R to B.

(a) Prove that F is a function. (Hint: Use properties of equivalence relations.)

(b) Prove that F is a bijection from A/R onto $\operatorname{Rng} f$. (Since F is a function, you may use the notation $F(x/R) = f(x)$ in showing this.)

2. Let $f\colon A \to B$ and $g\colon B \to C$ be functions. Prove that if $g \circ f\colon A \to C$ is injective then so is f.

3. Prove that a function $f\colon A \to B$ is a bijection if and only if there exists a function $g\colon B \to A$ such that $g \circ f = \mathrm{id}_A$ and $f \circ g = \mathrm{id}_B$.

4. Let A and B be finite sets having m and n elements, respectively. Consider the set B^A of all functions from A to B. Show that B^A has cardinality n^m by producing a bijection from B^A to the set

$$B^m = \underbrace{B \times \cdots \times B}_{m \text{ times}}.$$

Prove that your function is indeed a bijection. (This exercise explains the notation B^A.)

5. A *binary decimal expansion* is a list

$$0.a_1a_2a_3a_4\ldots$$

such that $a_i = 0$ or $a_i = 1$ for all $i \geq 1$. For example, such a list might look like

$$0.1001101001\ldots$$

Prove that the set of all binary decimal expansions is uncountable.