ASSIGNMENT 5, DUE WEDNESDAY 28 FEBRUARY

1. Let A and B be sets. Prove that $A \cap B = A$ if and only if $A \subseteq B$.

2. Let U be a set. Recall that we can define 'addition' and 'multiplication' on the power set $\mathscr{P}(U)$ by the rules

$$A + B = (A - B) \cup (B - A) \qquad \text{and} \qquad AB = A \cap B.$$

Let $U = \{1, 2, 3\}$. Write down the addition and multiplication tables for $\mathscr{P}(U)$.

3. Let A be a set and $\{B_\alpha \mid \alpha \in \Delta\}$ an indexed family of sets. Show that

$$A - \Big(\bigcap_{\alpha \in \Delta} B_\alpha \Big) = \bigcup_{\alpha \in \Delta} (A - B_\alpha).$$

4. Let $\mathscr{A}$ be the empty family of subsets of a set A. Prove that $\bigcap_{X \in \mathscr{A}} X = A$.

5. A *decreasing family of nested sets* is a family $\{A_i \mid i \in \mathbb{Z}^+\}$ such that $A_j \subseteq A_i$ if $i \leq j$.

(a) Show for all $n \in \mathbb{Z}^+$ that $\bigcap_{i=1}^{n} A_i = A_n$.

(b) Show that $\bigcup_{i=1}^{\infty} A_i = A_1$.

6. We say that a decreasing family of nested sets is *strictly decreasing* if $A_i \neq A_j$ for $i \neq j$. Construct a strictly decreasing family of nested subsets of $\mathbb{R}$ such that

$$\bigcap_{i=1}^{\infty} A_i = \{0, 1\}.$$

Justify your answer.