

ASSIGNMENT 7, DUE FRIDAY 30 MARCH

For the following problems, let F_n denote the n th Fibonacci number.

1. Let a be a fixed natural number. Use complete induction to prove that

$$F_a F_n + F_{a+1} F_{n+1} = F_{a+n+1}$$

for every natural number n .

2. Let α be the positive root and β the negative root of the polynomial $x^2 - x - 1$. Use complete induction to prove that

$$F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$$

for every natural number n .

3. Write a completed version of Monday's group activity.

4. Let n be a positive integer and $0 \leq r \leq n$. Prove that

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}.$$

5. In how many ways can n people be seated around a (round) table? Here we consider two arrangements to be the same if, for each person, the person to their left is the same in both arrangements. Explain your reasoning.