ASSIGNMENT 8, DUE FRIDAY 6 APRIL

1. Let A and B be finite sets. Prove that $|A - B| = |A| - |B|$ if and only if $B \subseteq A$.

2. Let n be a positive integer and let $1 \leq i \leq n$. How many permutations

$$a_1, a_2, \ldots, a_n$$

of the numbers $1, 2, \ldots, n$ satisfy the property $a_i = i$? (Such a permutation is said to *fix* i.)

3. A *derangement* is a permutation $a_1, a_2, \ldots, a_n$ of the numbers $1, 2, \ldots, n$ such that, for each $1 \leq i \leq n$, $a_i \neq i$. For example, $3, 1, 2$ is a derangement of $1, 2, 3$, but $3, 2, 1$ is not, because $a_2 = 2$. Find the number of derangements of $1, 2, 3, 4$. (Hint: Let X denote the set of all permutations of $1, 2, 3, 4$, and for each $1 \leq i \leq n$, let X_i denote the set of all permutations that fix i. Then the set of all derangements is

$$X - (X_1 \cup X_2 \cup X_3 \cup X_4).$$

Use the previous two questions along with the principle of inclusion/exclusion.)

4. Suppose that you are at the origin of the x-y-plane in a vehicle that is only capable of moving one unit up, or one unit to the right. Let n be a natural number. In how many ways would your vehicle enable you to reach the point (n, n)?

5. Let n be a natural number. Prove that

$$\sum_{i=0}^{n} \binom{n}{i} (-1)^i = 0.$$